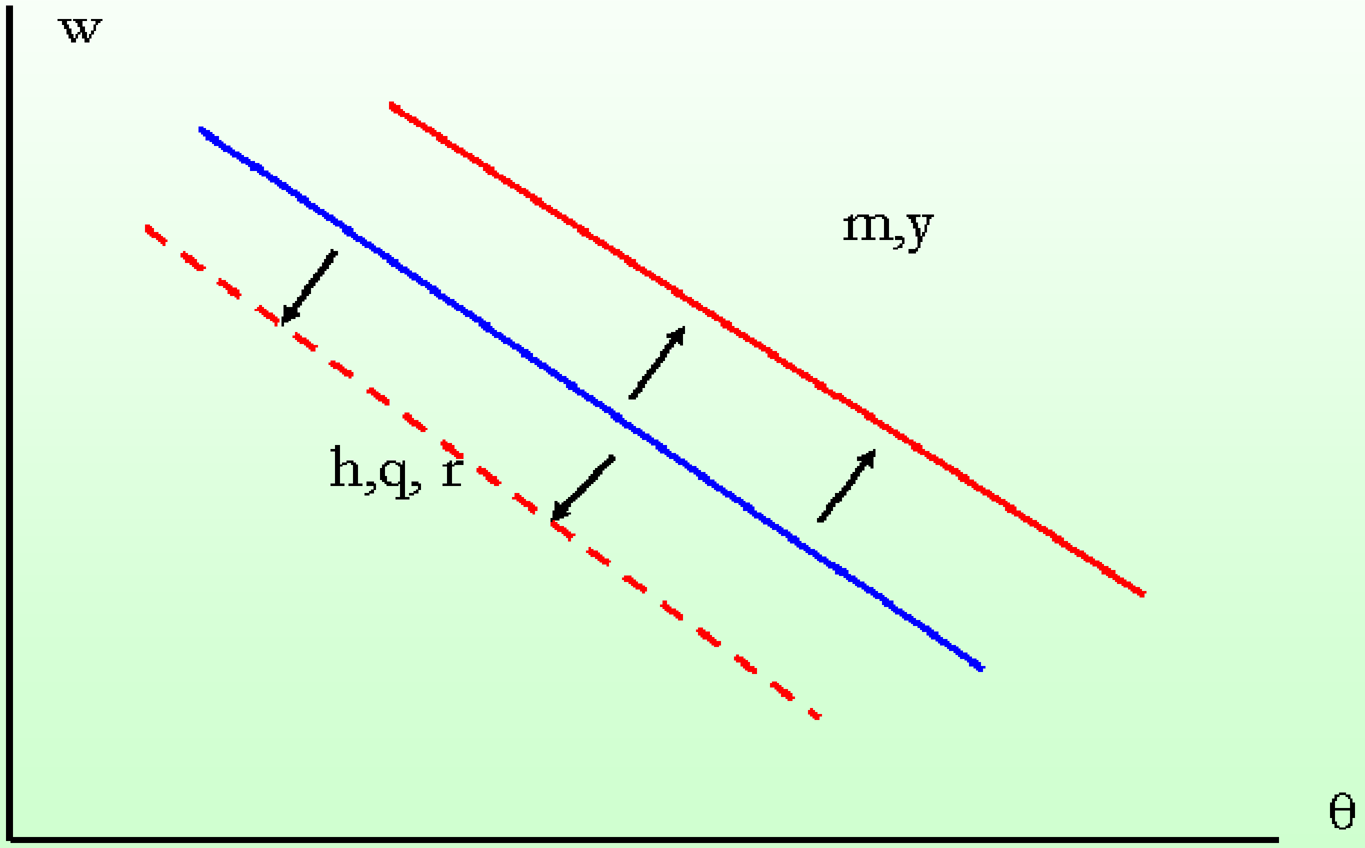
The Matching Model – Firm behaviour

- Remember labour demand: $[h/m(\theta)]=(y-w)/(q+r)$.
  - This can also be expressed: $w=y-h(r+q)/m(\theta)$, where $dw=h(r+q)\,[m'(\theta)/m(\theta)^2]d\theta$.
- Implications:
  - Increased interest rate shifts the labour demand equation negatively,
  - Increased job destruction rate shifts the labour demand equation negatively,
  - Increased costs associated with keeping vacancies open shifts the labour curve negatively,
  - Increased productivity shifts the labour curve positively.

The Matching Model – Firm behaviour



The Matching Model – Worker behaviour

- Basic assumptions:
  - Fixed infinity-living risk neutral individuals N.
  - Any worker can either be employed, receiving expected utility V_e, or unemployed, receiving expected utility V_u. $V_e \geq V_u$.
  - When employed he or she produces quantity y, and receives real wage w per unit of time.
  - As unemployed your net benefits are z every moment of time.
- The expected utility of employment:
  - $r V_e = w + q [V_u - V_e]$
- The expected utility of unemployment:
  - $r V_u = z + \theta m(\theta)[V_e - V_u]$ (remember: $\theta m(\theta)$ = exit from U)

The Matching Model – Worker behaviour

- 1)Define the global surplus from the match as:
  - $S=[V_e-V_u]+[\Pi_e-\Pi_v]$ {rent for the worker, rent for employer)

- 2)Nash bargaining/sharing rule defining how the global surplus should be divided:
  - $Max_w\ [V_e-V_u]^{\gamma}[\Pi_e-\Pi_v]^{(1-\gamma)}$
  - $\rightarrow (1-\gamma)[V_e-V_u]+\gamma[\Pi_e-\Pi_v]=0 \rightarrow \gamma[\Pi_e-\Pi_v]+S-[\Pi_e-\Pi_v]-\gamma S-\gamma[\Pi_e-\Pi_v]=0$
  - $[\Pi_e-\Pi_v]=(1-\gamma)S$
  - And consequently then $[V_e-V_u]=\gamma S$.

The Matching Model – Worker behaviour

- 3)Derivation of the negotiated wage:
  - Find an explicit expression for the global surplus:
  - Since i)$S=[V_e-V_u]+[\Pi_e-\Pi_v]$, ii) $r V_e=w+q[V_u-V_e]$ and iii) $r\Pi_e=y-w+q(\Pi_v-\Pi_e)$ then $S=[y-r(V_u+\Pi_v)]/(r+q)$
  - But $r\Pi_e=y-w+q(\Pi_v-\Pi_e) \leftrightarrow (r+q)\Pi_e=y-w+q\Pi_v -r\Pi_v + r\Pi_v$ $\rightarrow \Pi_e-\Pi_v=[y-w-r\Pi_v]/(r+q)$
  - Similar technique for the workers utility give: $V_e-V_u=[w-rV_u]/(q+r)$
  - But now we can use the sharing rules, and the fact that profits from vacancies are zero in equilibrium (free entry).

The Matching Model – Worker behaviour

- The negotiated wage:
  - $w = rV_u + \gamma(y - rV_u) = (1 - \gamma)rV_u + \gamma y$

- Interpretation:
  - If $\gamma = 1$ then the employee has all the power and reaps all the value from production y
  - If $\gamma = 0$ then the employer has all the power and reaps all the value from production y ($w = rV_u$ and $V_u = V_e$)
  - A linear combination of the value of production and the reservation wage, rV_u, with the respective bargaining power parameters as weights.

The Matching Model – Worker behaviour

- Our target is now to derive the wage curve (expressing labour supply and linking labour market tightness to wages):
- Since $r\ V_u = z + \theta\ m(\theta)[V_e - V_u]$ and $[V_e - V_u] = \gamma S$ (from the bargaining process) then $rV_u = z + \theta\ m(\theta)\gamma S$

- But we also know the value of the surplus at free entry equilibrium is given by $S = [y - r(V_u + \Pi_v)]/(r+q)$, (and remember free entry $\Pi_v = 0$),
  - Thus $rV_u = [(r+q)z + \gamma y\ \theta\ m(\theta)]/[(r+q+ y\ \theta\ m(\theta)]$

- But since $w = rV_u + \gamma(y - rV_u)$ we can solve for rV_u and plug in above, thus finding a relationship between wages and labour market tightness (as expressed by θ):
  - $w = z + (y-z)\Gamma(\theta)$, where $\Gamma(\theta) = \gamma[(r+q)z + \theta\ m(\theta)]/[(r+q+ y\ \theta\ m(\theta)]$

The Matching Model – Worker behaviour

- The wage curve: $w = z + (y-z)\Gamma(\theta)$,

  where $\Gamma(\theta) = \gamma[(r+q)z + \theta\, m(\theta)]/[(r+q + y\, \theta\, m(\theta)]$ and $\Gamma'(\theta) > 0$

  - Note: $\Gamma(\theta)$ represents the actual weight of the employee in the bargaining.
  - If θ increases, the probability of leaving unemployment increases, and then the opportunity value V_u increases, causing less worry over unemployment, and then the bargaining strength of the employee increases.
  - The opposite story provides the reason why increased job destruction and interest rate lowers $\Gamma(\theta)$ (reduced opportunity value V_u).

The Matching Model – Worker behaviour

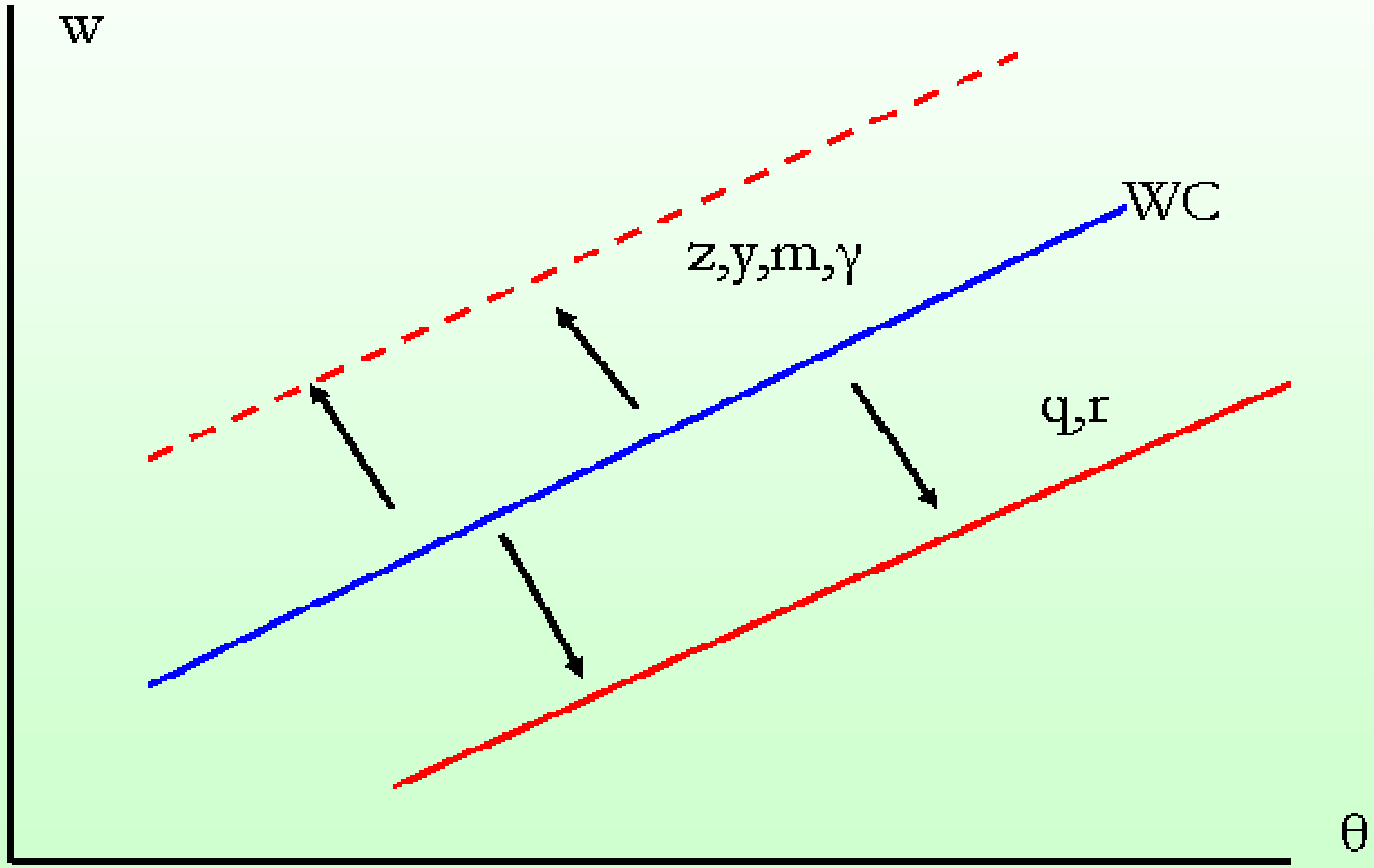
- The wage curve: $w = z + (y-z)\Gamma(\theta)$,

  where $\Gamma(\theta) = \gamma[(r+q)z + \theta\, m(\theta)]/[(r+q+ y\, \theta\, m(\theta)]$ and $\Gamma'(\theta)>0$

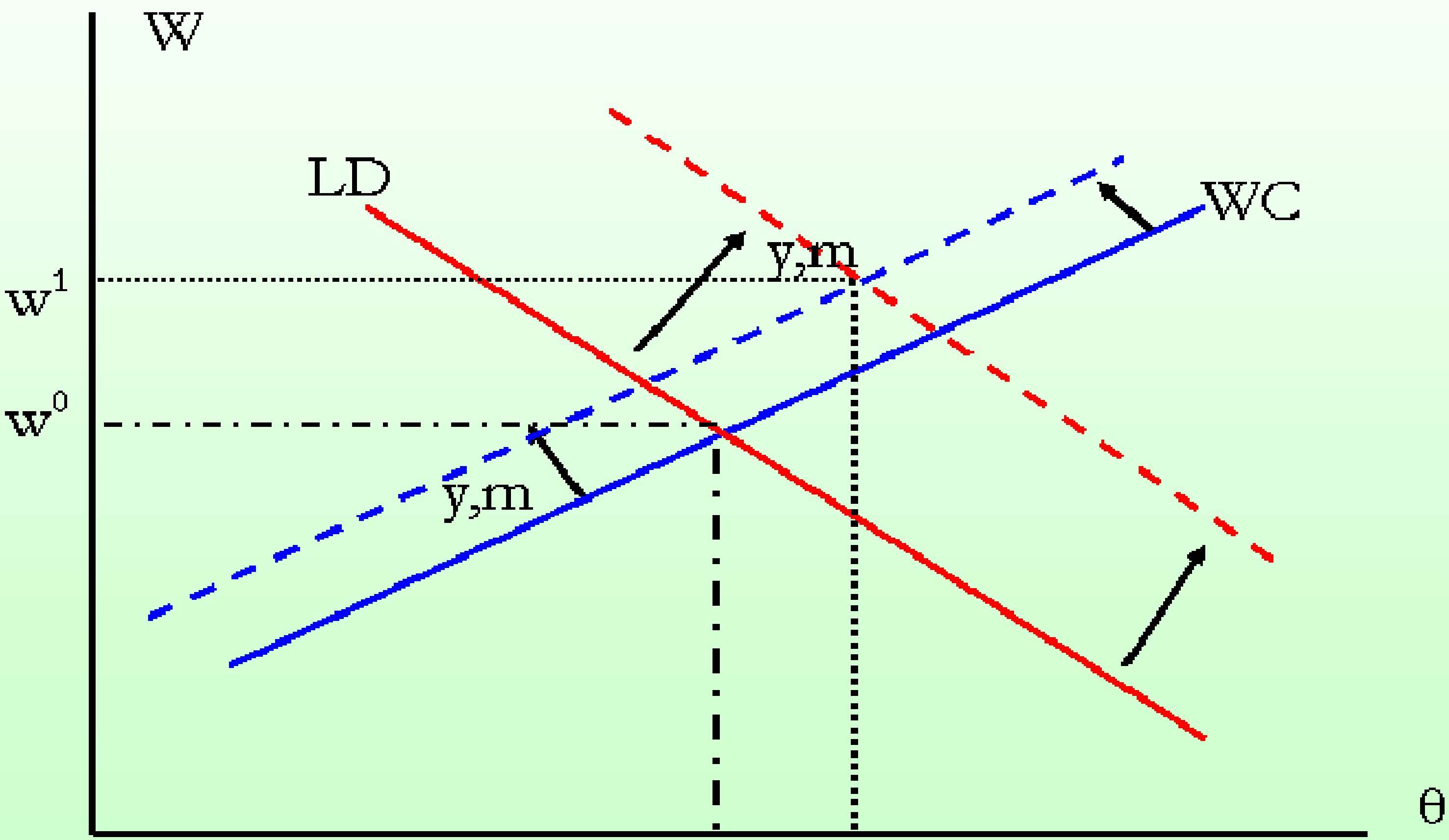
  - Increased net benefits, increased matching effiency, increased productivity and increased bargaining power all shifts the wage curve upwards (through increased opportunity value V_u).

  - Increased job destruction or increased interest rate lowers the wage curve (through reduced opportunity value V_u).

The Matching Model – Worker behaviour



The Matching Model – Equilibrium



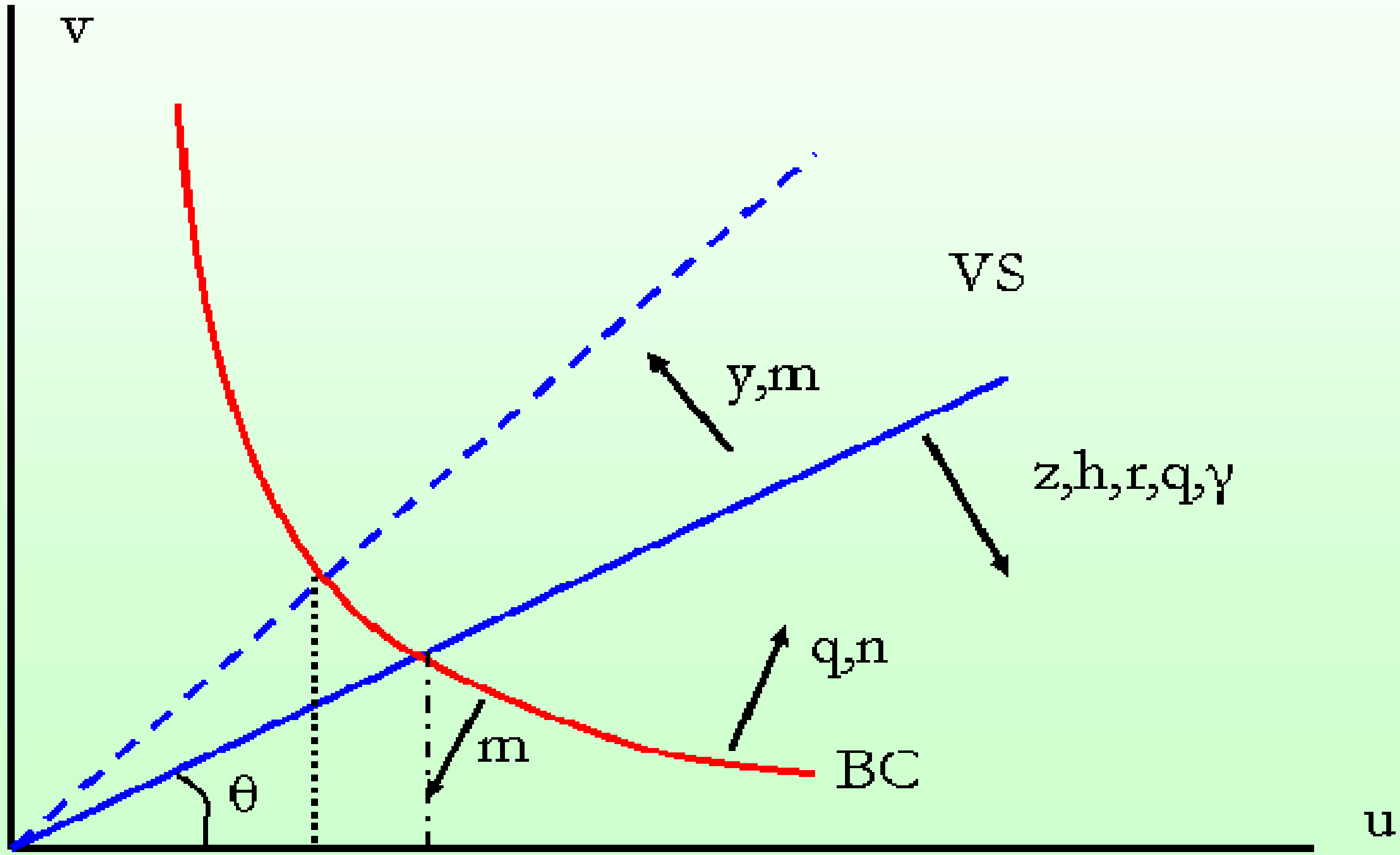
The Matching Model – Equilibrium

- In equilibrium we can combine the labour demand curve and the wage curve, and thus get rid of w. The following relationship uniquely solves the labour market tightness as a function of the exogenous parameters y, z, r, q, γ, h and m: $[h/m(\theta)]=[(1-\gamma)(y-z)]/[(r+q+ y\ \theta\ m(\theta)]$

- Left-hand-side: value of the expected profit from a filled job when one takes into account bargaining.
- Right-hand side: average cost of a vacant job

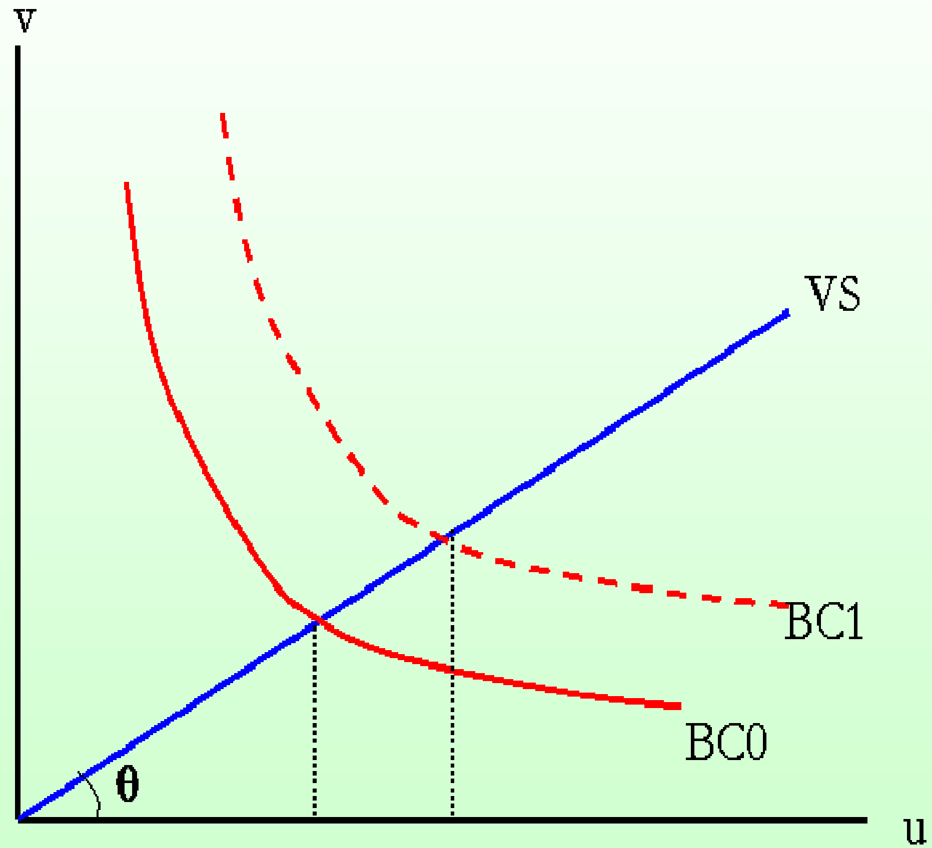
  - Note: since $d\theta=(1/u)dv-(v/u^2)du$ then if $d\theta=0$ we find the slope of curve expressing labour market tightness starting in origo as $dv/du=v/u= \theta$.

The Matching Model – Equilibrium

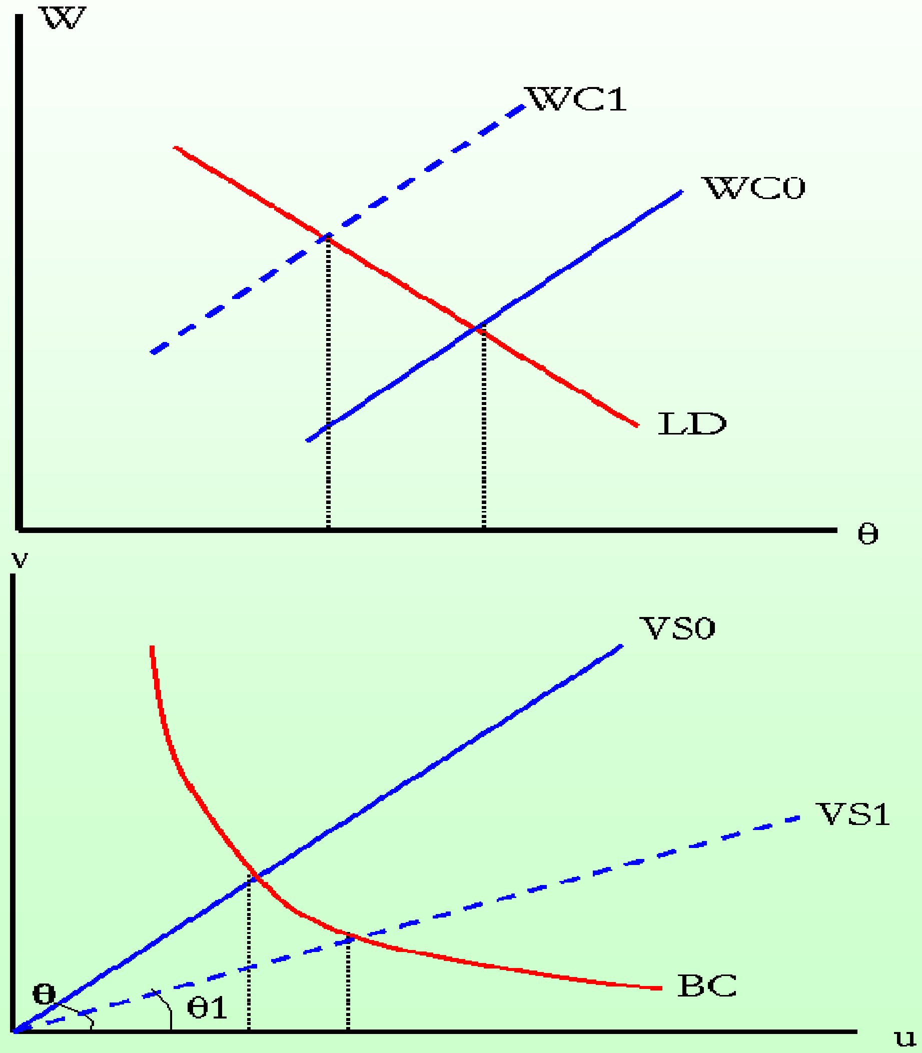


The Matching Model – Labour force growth

- Nothing happens with WC and LD
- All starts as unemployed, so more competition about vacancies
- BC shifts outwards
- Labour force growth equals deterioration of the matching process

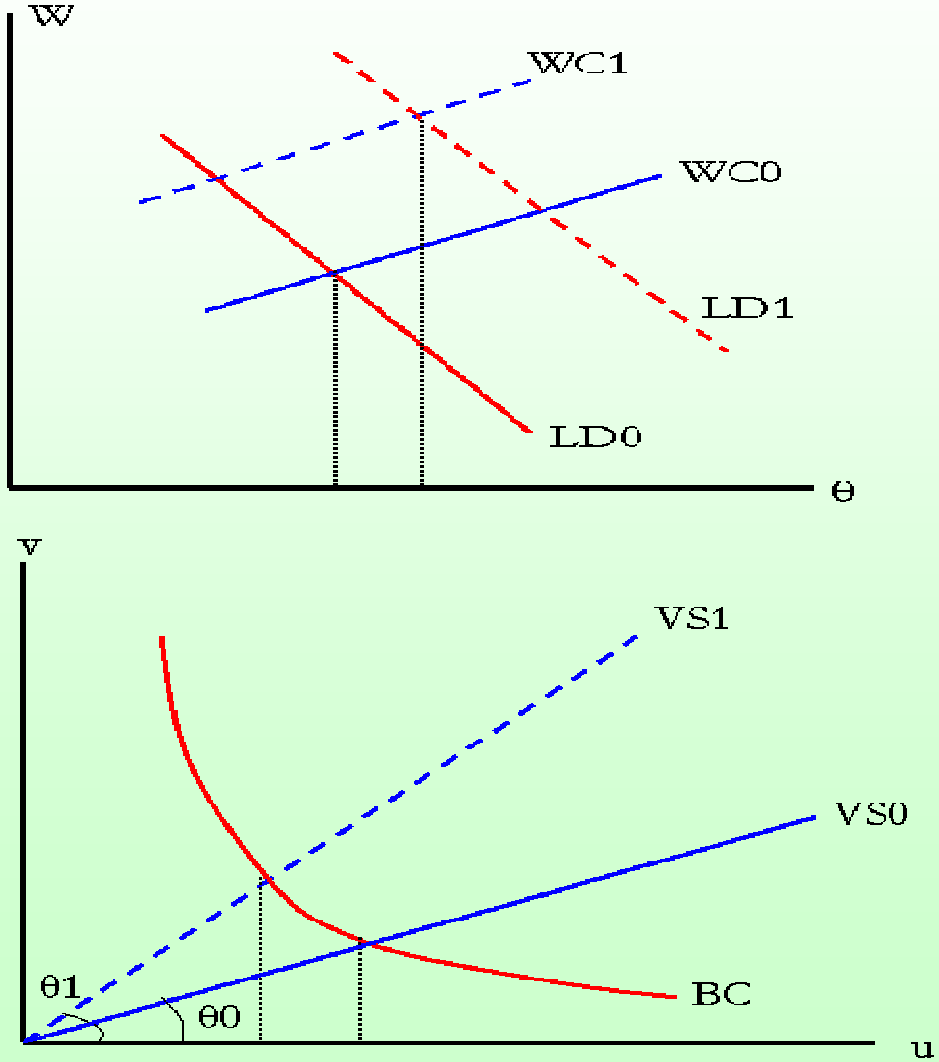


The Matching Model – Increased bargaining power or increased unemployment benefits



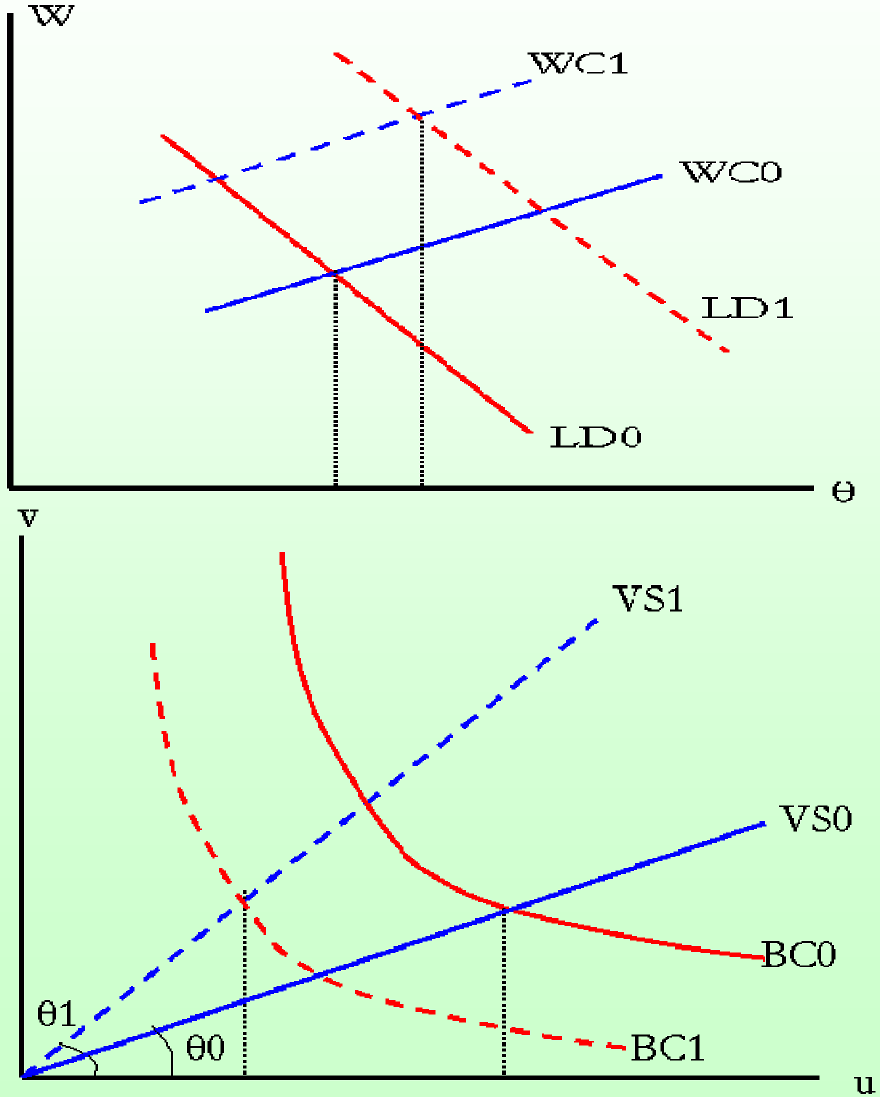
- Increased bargaining power increases wage costs, and Increased UB increases worker power in bargaining, so WC shift upwards
- Reduced profits from filled jobs
- Reduced entry of vacancies
- θ drops
- BC unchanged, v falls while u increases

The Matching Model – Increased individual productivity



- A rise in y increases total revenue, increasing both profits and wages. Both LD and WC shift upwards.
- Increased wages reduce the supply of vacancies
- Increased profits increase the vacancy supply
- The latter dominates, since profit from a filled job always increases with labour productivity
- θ increases, vacancy rate grows and unemployment drops

The Matching Model – Increased matching efficiency or reduced job destruction rate



- A rise in m increases the work probability, thus increasing worker bargaining stance and shifts WC upwards. Fall in q likewise since less change of becoming unemployed increases worker stance.
- A rise in m makes it more probable of filling a vacancy, reducing average cost. Fall in q increases profits from filled job. LD shifts upwards.
- LD dominates, BC shifts inwards.
- θ increases, vacancy rate grows and unemployment drops

The Matching Model – Equilibrium

	z	γ	h	m	y	q	r	n
w	+	+	-	+	+	-	-	0
θ	-	-	-	+	+	-	-	0
u	+	+	+	-	-	+	+	+

The Matching Model – Extensions

- Introduction of capital.
- Efficiency of the market equilibrium
- Technological progress and productivity growth
- Creative destruction

The Matching Model – Introduction of capital

- Firm i utilises L_i and K_i to produce $F(L_i,K_i)$. Let $k_i=K_i/L_i$ and $f(k_i)= F(L_i,K_i)/L_i$

- $F_K(L_i,K_i)=r+\delta=f'(k)$

  Marginal productivity of capital=interest rate+capital depreciation

- $F_L(L_i,K_i)=w_i+h(r+q)/m(\theta)=f(k)-kf'(k)$

  Marginal productivity of labour=wage+adjustment costs

- Comment:
  - marginal productivity of labour completely determined by $r+\delta$, thus motivating why y in our basic model is constant.
  - Different kinds of capital may give rise to hold-up problems.

The Matching Model – Efficiency

- Easy to assume that the market solution creates an efficient solution in this model.
- Alas, this is in most cases not true. Trading externalities: congestion effects within groups and positive between group externalities.
- Only if the value of the employees bargaining power is equal to the elasticity of the matching function with respect to unemployment will this be true (Hosios condition).
- More elaborate wage mechanisms may solve the problem. But: ”At the present time, the efficiency of decentralised equilibrium remains and open question.” (C&Z,2005:556)

The Matching Model – Technological progress

- In the basic model individual productivity level increase caused reduced unemployment. This hinges on the assumption of fixed unemployment benefits and fixed vacancy costs.
- Increased labour productivity level may however also affect these. If so, then the reduction in unemployment is nulled out.
- Short-term positive impact vs. no long-term impact
- Changed productivity growth is something very different.

The Matching Model – Technological progress

- What about changed productivity growth following technological progress?
- Capitalisation effect: Technological progress improves labour productivity and thus increases the profit due to job creation.
- Which jobs are affected by technological progress is essential. Completely different scenario depending on whether all reaps the benefit of progress or just the newly created jobs.

The Matching Model – Technological progress

- Assume that job destruction rate is exogenous and all jobs benefit from technological progress.
  - Increased productivity growth rate acts as a reduction in the effective interest rate.
  - Wages increase, but unemployment is reduced.

- Assume that job destruction rate is endogenous and only the recently created job produces at the current maximum productivity. As time goes by this unit's productivity slumps.
  - Vintage perspective. Creative destruction.
  - Increased productivity growth yields shorter lifespan of an already existing job and reduces the exit rate from unemployment, thus unemployment rises.

The Search and Matching Model – Empirical controversy

- Not supportive:
  - Tripier (2003): The framework has problems explaining the contra-cyclical unemployment rate.
  - Wong (2003): Unable to explain the development of wage dispersion in the US.
  - Shimer (2005):the predicted fluctuations in unemployment and vacancies in response to plausible shocks are much too small. “the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average productivity, while the search model predicts that the two variables should have nearly the same volatility” (Shimer, 2005:25). The source of the problem is how one models the wage determination, where a bargaining solution yields a pro-cyclical wage absorbing the shock. Thus virtually no propagation is exhibited by the model.
  - Hall (2005): Only the introduction of sticky wages or wage rigidities is needed to provide satisfactorily business cycle properties for matching models.

The Search and Matching Model – Empirical controversy

- Weakly supportive:
  - Cole and Rogerson (1999): (Only) if one assumed that the average duration of unemployment lasted 9 months or longer, could the model account for the business-cycle facts.

- Strongly supportive:
  - Mortensen and Nagypál (2006):Others overemphasize the need for wage rigidity to explain the data on labor market fluctuation"
  - Mortensen and Nagypál (2006):"the model matches the volatility of the job-finding rate if the opportunity cost of continuing a job-worker match is high enough, where this opportunity cost should include both workers' opportunity cost of employment and turnover cost"